vector parallel to the basal plane and T_2 to the shear mode with displacement vector normal to the basal plane as shown in Fig. 1. Propagation along the hexad axis yields c_{33} and an internal check on c_{44} as shown in the following equations

$$\rho v_L^2 = c_{33} \quad (11)$$

and

$$\rho v_T^2 = c_{44} \quad (12)$$

The last of the five independent elastic constants c_{13} can only be derived from the equations for propagation at some angle between the hexad axis and the basal plane. For propagation at 45° to the hexad axis one obtains for the T_1 mode

$$\rho(v_{T_1})^2 = \tfrac{1}{4}(c_{11}-c_{12}+2c_{44}) \quad (13)$$

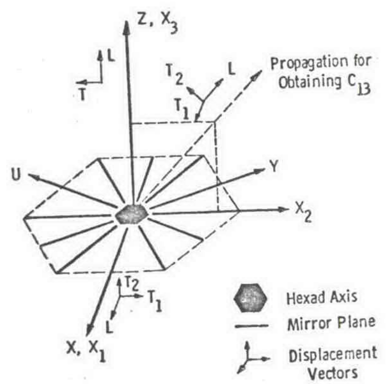

FIG. 1. Hexagonal ZnS symmetry element.

and for the L and T_2 modes

$$\rho v^2 = \tfrac{1}{4}(c_{11}+c_{33}+2c_{44}) \pm \tfrac{1}{4} \times \{(c_{11}-c_{33})^2+4(c_{13}+c_{44})^2\}^{1/2} \quad (14)$$

In this equation the positive second term applies to the L mode and the negative to the T_2 mode.

The equations for the curves of intersection of the velocity surfaces of the three acoustic modes with any plane containing the hexad axis can be obtained from equation (1) into which the computed values of the c_{ij} have been substituted. The appropriate equation is

$$[H-\tfrac{1}{2}c(1-n^2)] \times [H^2-\{n^2h+(1-n^2)a\}H+n^2(1-n^2) \times (ah-d^2)] = 0 \quad (15)$$

where n is allowed to assume all values between $+1$ and -1.

3. VELOCITY MEASUREMENTS

A single crystal of hexagonal ZnS, grown in these Laboratories, was cut and polished to have pairs of parallel faces normal to the X_1 and X_3 axes and to a direction in the X_2X_3 plane at 45° to either of these axes as shown in Fig. 1. The directions of the displacement vectors are shown in this diagram for each propagation direction used in these measurements. The transducers used were 10 mc/sec x- or y-cut quartz plates obtained from Valpey Crystal Corp. The pulse/cw technique used has been described elsewhere.[(2)] Table 1 lists the eight independent velocity measurements used for calculating the elastic constants given in Table 2 and the curves of intersection of the velocity surfaces with any plane containing the Z or X_3 axis shown in Fig. 2.

Table 1. Velocity measurements

Mode	Propagation direction (along axis)*	Displacement direction (along axis)*	Velocity $\times 10^5$ cm sec^{-1}
L	X_3	X_3	5·868
T†	X_3	X_1	2·645
L	X_1	X_1	5·667
T_1	X_1	X_2	2·815
T_2	X_1	X_3	2·644
L	45° to X_3	43° to X_3	5·469
T_1	45° to X_3	X_2	2·717
T_2	45° to X_3	−47° to X_3	3·224

* See Fig. 1.
† Shear modes degenerate.

Table 2. Elastic constants in units of 10^{12} dyn cm^{-2}

c_{11}	c_{12}	c_{44}	c_{33}	c_{13}
1·312	0·663	0·286	1·408	0·509

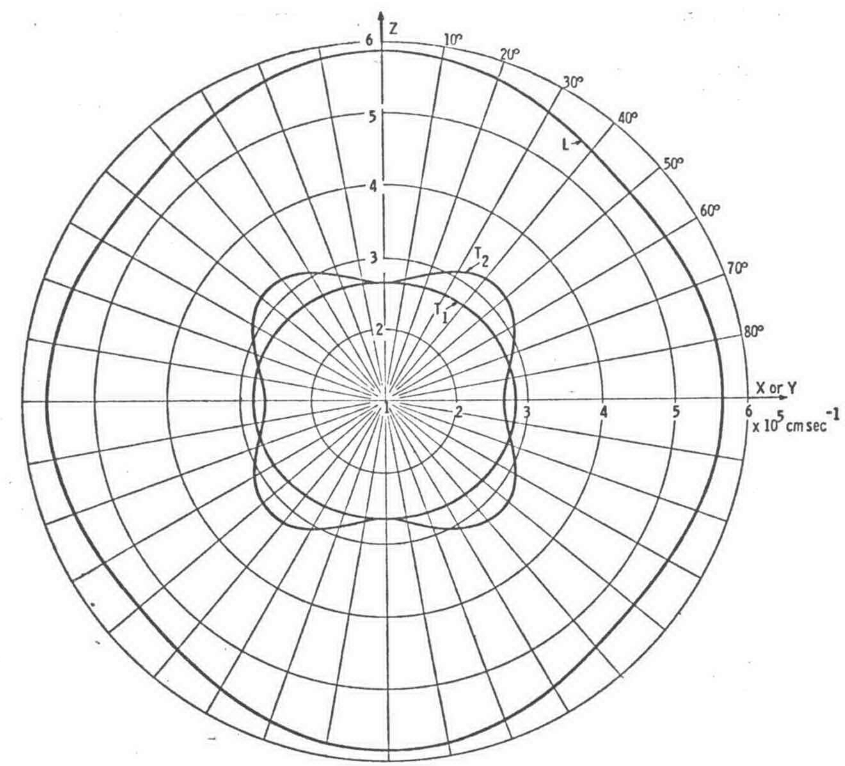

FIG. 2. Curves of intersection of velocity surfaces with any plane containing the Z axis.

4. WAVE SURFACES

The curves of intersection of the wave surfaces[(3)] with any plane containing the Z axis are loci of points R such that

$$R_i^2 = \frac{(v_i-A_i')^2}{(\cos\epsilon_i)^2}+2A_i'v_i-A_i'^2$$

or

$$R_i^2 = \left(\frac{H_i}{(\rho v_i\cos\epsilon_i)}\right)^2+2A_i'v_i-A_i'^2 \quad (16)$$

where

$$i = L,\quad T_1,\quad T_2$$

$$\cos\epsilon_i = \left\{\frac{m^2n^4d^4}{[(H_i-m^2a)^2+m^2n^2d^2]^2} + \frac{(H_i-m^2a)^4}{n^2[m^2n^2d^2+(H_i-m^2a)^2]^2}\right\}^{-1/2} \quad (17)$$

and

$$A_i' = \frac{c_{44}}{\rho v_i} \quad (18)$$

and H_i is as defined in equation (6). The parameters R_i, ϵ_i, A_i' and v_i are as defined in Fig. 3. The angle Δ between the wave normal and the direction of energy flow is defined by

$$\tan\Delta_i = \left(\frac{v_i-A_i'}{v_i}\right)\tan\epsilon_i \quad (19)$$

Figure 4 shows how the ray direction, or energy flow, deviates from the wave normal for each of the modes L, T_1 and T_2 as a function of θ, which is the angle between the Z axis and the wave normal, in any plane containing the Z axis. Figure 5 shows a plot of $(\Delta+\theta)$ as a function of θ for all three modes. The section of the T_2 mode curve for $20° < \theta < 70°$ corresponds to the cusp